Computer Simulation of 2-D and 3-D Diffusion Limited Aggregation

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A modified version of the Witten–Sander model for diffusion-limited aggregation (DLA) is proposed. Particles are launched one at a time from a randomly-selected point on an inner equal-stepped or equal-distant boundary. They undergo a random walk on the square lattice until they either stick to the cluster or ramble outside the outer fence. As the cluster is growing up, the inner and outer boundaries are expanded equivalently. The relation between the maximal size of cluster, the inner and the outer boundaries is \( R_{\text{max}} : R_{\text{max}} + R_{\text{in}} : R_{\text{max}} + R_{\text{out}} \). By changing the \( R_{\text{in}} \) and \( R_{\text{out}} \) scale, only a negligible variation of the fractal dimensionality is observed without changing the essential nature of fractals in DLA. Results from 3-D DLA simulations are also presented. A final trial is to adopt a moving centre model in 2D and 3D simulations. A satisfactory correlation between Witten-Sander model and Eden model have been constructed. Some clues about the aggregation processes may be revealed from the analyses of the DLA model with a moving seed.

Introduction

The aggregation of particles to form larger structures is important in many natural\(^1\)–\(^5\) and industrial\(^6\)–\(^8\) processes and is also a major research activity in the colloid and interface science.\(^9\)–\(^10\) In many cases, the structures formed by the far-from-equilibrium growth phenomena have a fractal-like geometry.\(^11\)–\(^13\) Examples for such processes include dendritic solidification in an undercooled medium,\(^14\) electrodeposition of ions onto an electrode\(^15\) and crystalline structure of snowflake.\(^16\) Much efforts have been made in order to get insight into the relationship between fractal structures and aggregation mechanisms.\(^17\)–\(^20\) Computer simulations have played an indispensable role in our comprehension about the aggregation patterns\(^21\)–\(^24\) for its convenient capacity to test ideas concerning the behavior of experimental systems and to pose well-defined problems for theoretical analysis. Furthermore, the information accumulated from the computer simulations with the minimal changing variables and maximal reproducibility can also be beneficial to experiments.

Recently, considerable interest in non-equilibrium growth processes has been generated by DLA growth model proposed by Witten and Sander in 1981.\(^25\) It is an example of randomly-walked particle producing beautiful self-similar clusters. For the case of 2-D DLA, a square lattice is used with a seed particle inside. Particle is then released one at a time from the perimeter of a large circle whose centre is coincident with the seed particle. The particle wanders a long way in Brownian-like motion until it either leaves the outer circle and restarts to save the computer time or enters into the neighboring region of the seed particle and becomes part of the DLA. This process is repeated several thousand times until a large cluster is formed. Although the growing process is simple, it gives rise to ramified, self-similar
and scale-invariant structures. A lot of strategies have been suggested to decrease the computer simulation time and to make a connection between the Witten-Sander model and other models. It shows that the fractal dimension $D_f$ is insensitive to the lattice type and sticking probability. Some generalizations are fabricated to associate the Witten-Sander model for diffusion-limited aggregation with other models. For example, by changing the step length exponent in Levy flight trajectories from zero to infinity, one can get continuous results from Vold-Sutherland linear trajectory model to Witten-Sander random-walk trajectory model.

In this paper, a modified version of Witten-Sander model is proposed to reduce computer time needed in the simulation by adopting an expandable inner and outer margins. Each time a particle is launched from a randomly-selected point on the inner boundary. It undergoes a random walk on the square lattice until it either sticks to the cluster or wanders outside the outer circumference. In 2-D simulations, square launching boundary is employed in addition to the circular one. Particles launched from a square boundary are equi-stepped from the seed particle, while particles launched from a circular one are equi-distant from the seed particle. In the 3-D simulations, the equal-stepped is of octahedral shape and the equal-distance one is a sphere. The effect of launching boundary shape on DLA will be discussed later on. As the cluster is growing up, the inner and outer boundaries are assumed to expand equivalently. The relation between the maximal size of cluster, the inner perimeter and the outer perimeter is $R_{max} : (R_{max} + R_{in} : R_{max} + R_{out})$. We have investigated the effect of the ratio and scale of $R_{in}$ and $R_{out}$ on fractal dimension and merely an insignificant deviation can be drawn from the results. A final trial is to use a moving centre model in 2D and 3D simulations. Here the centre of the inner and outer boundaries is randomly selected from the growing cluster and the length of the inner and the outer boundaries is constant during the simulations. The centers of the inner and outer margins are re-selected when a new particle is about to launch. A more compact fractal structure owing to the penetrating effect will be formed as the cluster size is growing up. These aggregation processes can be surveyed by snapshots of growing cluster taken at different sizes using different colors. The relation of the fractal dimension and cluster size is also discussed in Sec. III. In Sec. II, a brief review of fractal properties and the definitions of some useful quantities are given. Algorithms, assumptions and coding methods are also described in Sec. II. Some 2D and 3D simulation results are then presented and discussed in Sec. III with figure illustrations. Some concluding remarks and future development are given in the Sec. IV.

**Methods and Models**

Fractal geometries are observed in the naturally-grown objects. The structures of fractals are found to possess some inherent symmetry. However, they are apparently random under closely investigation. In some sense, one can make crude assumption that the shape of any object tends to depend strongly on the kind of process from which the object originated. Questions arise when we want to quantify the form of these intrinsically symmetric objects. What is invariant during the growth process? And what are the mechanisms and parameters that control the growth process? Here we propose two mechanisms and by computer-simula-
tion we can obtain two fractal patterns. Some characteristics can be used to elucidate the essential features of these geometries. One common terminology is the fractal dimensionality \( D_f \). In Euclidean space, the density of any object is irrelevant to the length scale on which it is measured. Hence in two dimension space we have two independent directions and the area will be four times bigger if the length scale is doubled. However, in fractal dimension space, fractal is the naive feature that can be attributed to the decreasing density with increasing length scale. Therefore, as the length scale is \( \mu \) times bigger, the mass obeys the following relation

\[ M(\mu L) = \mu^n M(L) \]

Here \( D_f \) is a fractal dimensionality and is always smaller than the Euclidean dimension. The other criterion is the average ratio of neighboring number for each cluster. Owing to the self-similarity of the fractal geometries, the ratio of neighboring number for each DLA aggregate will be constant if the fractal dimensionality is constant during the aggregation process. If the \( D_f \) is varied, which means the self-similarity no longer exists, the ratio of particles having different nearest neighbors will be adjusted to a new balanced value. These informations are also strong manifestations in DLA fractal patterns. There are four nearest neighbors at maximum in two dimensional space and six in 3-dimensional space. If the fractal is compact, the proportion of particles having higher neighboring number will be elevated and vice-versa.

Our first model is a modified version of Witten–Sander model for diffusion-limited aggregation. As in the DLA model, a seed is assumed to be placed at the centre of the square lattice. Particle is launched one at a time from a randomly-selected point on a circle whose center is coincident with the seed particle. The selected particle undergoes a random walk until it either reaches an unoccupied lattice site neighboring to the growing cluster or it detours a long distance on the lattice until it goes out of the outer perimeter. In Witten–Sander model, particles which reach a large distance from the center (typically \( 3R_{\text{max}} \), where \( R_{\text{max}} \) is the maximal radius of the cluster) are annihilated and a new particle is started off from the launching circle. Usually the radius of the launching circle is \( R_{\text{max}} + 5 \) unit of lattice constant. There are many constraints to Witten–Sander model. Firstly, as the simulation started, the launching circle is too large to get an cluster of appreciable size in a finite CPU consumption. Secondly, the cluster size is constrained to \( R_{\text{max}} \). The accumulation process is terminated in their simulation when any branch of the cluster touches the margin \( R_{\text{max}} \). The outer region from \( R_{\text{max}} \) to \( 3R_{\text{max}} \) is desolated as a random walk district instead of an aggregate region. Thirdly, particles digress a fairly long distance to go out off the outer boundaries and so much CPU time are wasted in these procedures. Our model is to develop a compromising method between these two ends. An acceptable radii of launching circle and outer boundary, which vary as the cluster size grows, are adopted to optimize the cluster growth rate and minimize the CPU time consumption. When the cluster is growing up, the radii of these two boundaries expand correspondently. The relationship of the radius of the cluster, the launching circle, and the outer boundary is \( \frac{R_{\text{max}}}{R_{\text{in}}} = \frac{R_{\text{max}} + R_{\text{in}}}{R_{\text{max}} + R_{\text{out}}} \). In order to test the generality of our model, we have tried several sets of ratio and scale of \( R_{\text{in}} \) and \( R_{\text{out}} \). Insignificant deviations are observed in our simulations and the results are in good agreement with other experimen-
tual works\textsuperscript{33,34} and theoretical calculations\textsuperscript{35,36,39,35}. The success of this model can be attributed to the shielding of outer periphery of the cluster and the meandering of the randomly-walking particles. These two factors conspire to give the brilliant success of this model. These results will be presented in the next section.

Our second model is to introduce a method to construct a cluster with continuously increasing fractal dimensionality. As in the previous model, the center of the launching circle is coincident with the seed particle. However, in this model, the center is randomly chosen from the growing cluster each time a new particle started to fire but the radius of the launching circle and the radius of the outer margin are kept fixed. Here the particle is justified to be outside only when its distance (the least steps) to each particle in the cluster are all greater than the radius of the outer boundary and a new center are re-selected. When the cluster is growing up and becomes more compact, some lattice sites maybe occupied by the cluster particles and the launching particle could be one particle of the cluster. A new particle is re-selected until all the lattice sites of the launching circle are completely populated by the cluster particles. Under such situation a new center and thus a new launching circle is chosen to keep the aggregation process going. The fractal dimensionality and the ratio of particles having different nearest neighbors demonstrate the variations of aggregation patterns. The resulting fractals are more compact and may correlate to some aggregation or crystallization processes. Furthermore, the limiting case when extrapolated to zero cluster size is the Witten-Sander DLA aggregation model. These simulation results will also be discussed in the next section.

In addition to a circular perimeter which is used as the fence of launching circle and outside boundary, a square perimeter is also adopted as it is equi-stepped while the circular one is equi-distant from the seed particle. A comparison is illustrated in Figure 1. A strong motivation for adopting a square perimeter is the least steps to boundary are all equal in all directions but in the circular one the least steps are a function of angle and can be expressed as

$$D_{ls} = R(\sin(\theta/90') + \cos(\theta/90') | \mod(\theta/90'))$$

where

- $R$ is the direct-line distance from boundary to seed particle
- $\theta$ is the rotating angle from $x$-coordinate
- $\mod$ is the residue number

$D_{ls}$ has a maximum as $\theta$ equal to $45' + 90'x (x = 0,1,2,3)$. A particle released from a circular boundary will take more random walk time to either stick to the cluster or go outside the outer boundary, because there are more awkward abundance of lattice sites inside a circular boundary compared to the amount of lattice sites inside a square one. These slight differences will make some deviations in the results which will also be revealed in the next section.

In order to record all the lattice sites on the perimeter of the launching boundary, an integer array is opened to store all the coordinates of lattice sites between $R_{in} + R_{max}$ and $R_{in} + R_{max} - 1$ for our first model and $R_{max}$ and $R_{max} - 1$ for our second model. The launching particles are randomly chosen from this array. Another one-dimension array is used to regis-
ter the positions of the cluster particles in our second model and the centre of inner and outer boundary is re-selected from this array each time a new particle started to launch. A two-dimension array is used to record the situations of all square lattice sites in 2D simulation. The cells of this array labelled 0, 1, and 2 indicate a vacant site, an occupied position and one of the vacant nearest-neighbor positions surrounding the cluster, respectively. These informations are essential for the particle movement. Initially, only the position of the seed particle is labelled 1 and the four nearest-neighbor sites are marked 2. Particles standing on the sites labelled 0 are free to move randomly. When migrating into the sites marked 2, they become part of the cluster. These sites are re-labelled as 1 and the nearest-neighbor sites of the newly-formed cluster are re-labelled 2. These schemes are adopted in both 2-D and 3-D simulations.

In the next section, both 2-D and 3-D results are presented. We have carried out the simulations using these two mechanisms in both 2-D and 3-D space. Including the equal-stepped and equal-distant perimeters in each mechanism, totally, we have eight operating conditions. For example, in two dimension space, a 13000-particle cluster can be generated from simula-

<table>
<thead>
<tr>
<th>Model</th>
<th>Moving Seed Model</th>
<th>Fixed Center Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td>Equal-stepped</td>
<td>Equal-distant</td>
</tr>
<tr>
<td>2 D</td>
<td>32114</td>
<td>32108</td>
</tr>
<tr>
<td></td>
<td>3540</td>
<td>3540</td>
</tr>
<tr>
<td></td>
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<td>34-36 hr</td>
</tr>
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<td>3 D</td>
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<td>3540</td>
<td>3540</td>
</tr>
<tr>
<td></td>
<td>30-32 hr</td>
<td>32-34 hr</td>
</tr>
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</table>

a. numbers of particles
b. 3540: VAX station 3540 3600: μ-VAX 3600 8530: VAX 8530
c. CPU consumption time

**Figure 1.** A comparison between square and circular launching boundaries. A square margin is equal-stepped from the centre in all directions but a circular one is a function of angles.
Figure 2. A 2D 6000-particle cluster generated from fixed-center Witten-Sander model using (a)square and (b)circular launching boundary with colour alternation to indicate 1000-event increment.

Figure 15. Snapshots of a growing cluster generated in 2D simulation of moving seed model taken at size of (a)2000 (b)4000 (c)8000 (d)16000 particles. As can be seen, the fractal patterns are getting more and more compact with continuously increasing fractal dimensionality $D_f$ of (a)1.80 (b)1.83 (c)1.86 (d)1.92.

Figure 10. A typical resulting aggregate of 12000 particles generated from 2D simulations of moving seed model with colour alternation to indicate the 3000-event aggregation.
tion of fixed-centre model using square launching boundary. It takes about 2-hour CPU time on a VAX-8530 computer. The CPU time consumed in each machine is listed in Table I.

In the following sections, unless specified elsewhere, all the figure illustrations with two icons are divided into Figure (a) and Figure (b). Figs.(a) represent the results by projecting particles from equal-stepped launching boundaries and Figs.(b) are the results from equal-distant launching perimeters.

Results and Discussions

For the 2-D DLA aggregates of the modified Witten-Sander model, simulations were performed on a 700 × 700 square lattice with different shapes of launching boundaries. The resulting aggregates of 2-dimension simulations are displayed in Figure 2. Figure 2(a) shows the DLA aggregate of our first model generated by projecting the particles from a square boundary while Figure 2(b) from a circular one. Inasmuch as the mismatched distance in every direction from a circular perimeter, the launching sites at the right angles will be the least-stepped positions from the center and the most possible sites to accumulate statistically. Thus, a conspicuous four leaves and a square-like contour can be distinguished from Figure 2(b). On the opposite side, Figure 2(a) gives a relatively isotropic figure since the propagating rates are equal in all direction. A second difference between these two configurations with equal radius is that particles launched from circular boundary have larger probabilities to stay inside and will squander much random walk time owing to its clumsy occupancy of lattice sites (here the radius for square boundary is the steps to its centre). Hence particles projected from the square margin has the prevailing chances to adhere to the cluster. The different aggregation efficiency is shown in Figure 3 by displaying the relations between the

![Figure 3](image.png)

**Figure 3.** The aggregation efficiency depicted as the variation of cluster size vs. numbers of shots for 2D simulations of fixed center Witten-Sander model using (a)square and (b)circular launching boundary.
Table II. The fractal dimensionality under different ratio and scales of $R_{in}/R_{out}$ in 2D simulations.

<table>
<thead>
<tr>
<th>$R_{in}/R_{out}$</th>
<th>Boundary Type</th>
<th>Square</th>
<th>Circular</th>
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<tbody>
<tr>
<td>6/36</td>
<td></td>
<td>1.67</td>
<td>1.70</td>
</tr>
<tr>
<td>12/36</td>
<td></td>
<td>1.65</td>
<td>1.64</td>
</tr>
<tr>
<td>18/36</td>
<td></td>
<td>1.66</td>
<td>1.064</td>
</tr>
<tr>
<td>24/36</td>
<td></td>
<td>1.67</td>
<td>1.69</td>
</tr>
</tbody>
</table>

Figure 4. The variation of nearest-neighbor distributions as function of cluster size in 2D simulations of fixed-center Witten-Sander model using (a)square and (b)circular launching boundary. Here the sums of total ratio of nearest-neighbor in each cluster size should be amounted to 1. Filled squares, open circles, open diamonds and filled circles stand for ratio of particles with 1, 2, 3 and 4 nearest-neighbors in the growing cluster, respectively.

Figure 5. The fractal dimensionality calculated as every 2000 particles were added to the growing cluster in 2D simulations of fixed-center Witten-Sander model using (a)square and (b)circular launching boundary.
number of shots and the cluster size. Curve(b) for the square launching sites exhibits a more efficient aggregation rate. This can also be reflected in the CPU consumption times which are tabulated in Table I. It is obvious from row 2 and row 3 that square boundary is a more beneficial choice rather than the circular one.

In our simulations, the radius of launching boundary R and outer perimeter R are 24 and 36 units of lattice constant. We have investigated a series of $R_{in}$ and $R_{out}$ to test the flexibility of our model. As listed in Table II, only an insignificant deviation can be drawn from column 1 and column 2. These facts suggest that particles launched from the margin have a scarce opportunities to penetrate into the unfilled interior of the growing cluster, i.e., they are screened due to the self-avoiding nature imposed by the random walk. They can only attach to the periphery of the cluster. The randomly-chosen particles from the margin, the Brownian-like motion of the particle, and the edge of the cluster which prevents the invasion of separated particles all conspire to make the fractal dimensionality irrelevant to the launching margin as well as the ratio and scale of $R_{in}/R_{out}$. As can be seen in Figure 2 the colour alteration indicates the 1000-event aggregation process and the concentric fringes demonstrate that these particles are all accumulated to the periphery of the cluster. A final manifestation is the ratio of neighboring numbers and fractal dimensionality in the growing cluster. Analyses were performed for these two quantities as every 2000 particles added to the cluster and the results are presented in Figs. 4 and 5. At 2-dimensional space, there are 4 nearest neighbors of each particle at most. As can be seen from Figs. 4 and 5, the linearity suggested

**Figure 6.** A 3D 4941-particle aggregate generated from fixed-center Witten-Sander model using octahedral launching boundary. Three different shading backgrounds are used to demonstrate steric construction.

**Figure 7.** The maximal cross section cut from the 3D aggregate shown in Figure 6. A porous and discontinuous particle distribution indicates a twisted structure.
that these fractal patterns are self-similar and scale-invariant in cluster size ranges from a very small fractal to whatever length scale. Also only small differences can be seen from these two figures in spite of the shapes of the launching boundaries. The fractal dimension is about 1.67 in excellent agreement with theoretical value.26,29,34

Similar operations were performed in 3-D space in a cubic lattice. Figure 6 shows a resulting aggregate of 4941 particles generated from fixed-centre model using octahedral launching boundary. The grid lattice used is of order 100 x 100 x 100 with R_in / R_out = 24 / 36. Owing to the screening of the outer surface and the view bias of landscape in 3-D plot, the interior core of the cluster and the entire construction are not shown in this paper. A video monitor would be helpful to our visualization of the aggregation process in 3-D space. The maximal cross section, as can be seen in Figure 7, shows a porous and discontinuous particle distribution. In fact, the branches of these clusters are twisted and crooked, which can be ascribed to the fact that there are three independent directions for the random walk motion.

**Figure 8.** The variation of nearest-neighbor distributions as function of cluster size in 3D simulations of fixed-center Witten-Sander model using (a) octahedral and (b) spherical launching boundary. Here the sums of total ratio of nearest-neighbor in each cluster size should be amounted to 1. Filled squares, plus signs, open diamonds, open triangle, cross signs and open flipped-triangle stand for ratio of particles with 1, 2, 3, 4, 5 and 6 nearest-neighbors in the growing cluster, respectively.

**Figure 9.** The fractal dimensionality calculated as every 2000 particles were added to the growing cluster in 3D simulations of fixed-center Witten-Sander model using (a) octahedral and (b) spherical launching boundary.
Hence the isolated particles have superior chances to penetrate into the unfilled cluster core and a more tangled structure will be formed. The ratio of particles having different nearest neighbors and the fractal dimension are analyzed as every 2000 particles were added to the growing cluster and are depicted in Figs. 8 and 9. As in 2-D simulations, these fractal geometries are self-similar and scale-invariant. The elucidations and explanations are all the same as those for 2-D results. Some supplements are described in figure captions.

For 2-dimension and 3-dimension DLA with moving seed in our second model, simulations were also performed at a $700 \times 700$ square lattice and a $100 \times 100 \times 100$ cubic lattice, respectively. A typical resulting aggregate of 12000 particles from 2-D simulation is shown in Figure 10. The impetus behind this model is to fabricate a cluster with continuously changing fractal dimensionality and to find the variation of cluster patterns. As the cluster is growing up, the interior core are filled up gradually. The concentric fringes in Figure 10 are now more zigzag and more penetrating to the unfilled nucleus of the cluster. The limiting cases in both directions might be viewed as a cluster with a tight interior when extrapolating to infinite large cluster size and a cluster with no chance to attach to the interior core when extrapolating to

![Figure 11. The variation of nearest-neighbor distributions as function of cluster size in 3D simulations of moving seed model using (a)octahedral and (b)spherical launching boundary. Here the sums of total ratio of nearest-neighbor in each cluster size should be amounted to 1. Filled squares, plus signs, open diamonds, open triangle, cross signs and open flipped-triangle stand for ratio of particles with 1, 2, 3, 4, 5 and 6 nearest-neighbors in the growing cluster, respectively.]

![Figure 12. The fractal dimensionality analyzed as every 2000 particles were added to the growing cluster in 3D simulations of moving-centre model using (a)octahedral and (b)spherical launching boundary.]


zero cluster size. The former is analogous to an Eden cluster while the latter is a Witten–Sander DLA aggregate. Analyses of the distribution of particles having different nearest neighbors in the growing clusters were also performed and the results are presented in Figs.11 and 12. As can be seen, the above mentioned trends are obviously exhibited in the ratio of particles having different nearest neighbors and the fractal dimensionality in 3-D simulations. Owing to the propensity of being outside in the case of square boundary, particles are easy to accumulate in circular margin and thus a sharp variation in the ratio of particles having different nearest neighbors is observed in Figure 11(b). Comparing the left end of each corresponding curves in Figure 11 with Figure 8 and Figure 12 with Figure 9, they will drop to the same vicinity when extended to zero cluster size, which is a notable evidence that the limiting case of our second model is a Witten–Sander DLA aggregate. Similar results are observed in 2-D simulations. The ratio of particles having different nearest neighbors and the fractal dimension were analyzed at different cluster size and are depicted in Figs.13 and 14. When extended to zero cluster size, they show the same tendency for each corresponding curves as to drive to the proximity as in Figs.4 and 5. The changes of fractal patterns with fractal
dimensionality of our second model in 2-D simulations are further illustrated in Figure 15 with snapshots taken at 4 different cluster sizes. This fascinating model has built a way from Witten–Sander DLA aggregate to Eden model. A complete visualization of the aggregation process can be realized using computer graphics.

Finally, the fractal dimensions obtained from the simulations of our first model in 2-D and 3-D simulations are found to be in good agreement with previous experimental works and theoretical calculations. A square launching boundary in 2-D and an octahedral launching boundary in 3-D simulations seem to be a more efficient way to generate larger clusters instead of the conventional circular and spherical one without changing the fractal characteristics. Our second model somehow displays the same features in the Levy flight model in a random walk model. The continuously changing of the fractal dimension as the DLA cluster grows deserves to be closely investigated.

Conclusions

We have studied two models of aggregation processes in two and three dimensional space. Each model was simulated with a square and a circular boundary. Totally, we have eight operating conditions. Our first model, in fact, is a modified version of Witten–Sander model with more economical strategies and our second model has built a bridge from Witten–Sander DLA aggregate to Eden model. The simulation results are also in good agreement with other’s works. From our rudimentary data, a square boundary is superior to a circular one for its easy aggregation and more isotropic fractal contour because particles are prone to go outside and a new particle can start to launch again to save the random walk CPU time.

However, some constraints have restricted the cluster size and aggregation rate. First, we need more data collections to facilitate the precision of our measurements. Second, the VAX micro-computer has a frugal capacity for array storage. If we would like to generate a larger cluster, some intrigues have to be developed to extend the cluster size. Third, the variation rate of the fractal dimension of our second model is too rapid as the cluster is growing. A combination of our first and second model to slow down the increasing rate of fractal dimensionality is currently underway. By changing the weight of these two models, we can get a more dense fractal patterns with considerable size. These results will be published elsewhere.

References


二維與三維擴散決定聚集的模擬

羅永隆  楊文麗  李錫隆*

摘 要

我們提出一種有關擴散決定聚集 Witten-Sander 模型的改進方法，粒子每次一顆從一等步或等距離的內邊界隨機射出並於方格點上任意行走直至其附著於中心粒子圖上或是跳出外邊界為止。隨著中心粒子圖的成長，內外邊界也隨之擴張。中心粒子圖的大小與內外邊界的關係為 $R_{\text{max}} : R_{\text{in}} + R_{\text{out}}$。我們發現改變 $R_{\text{in}}$ 和 $R_{\text{out}}$ 的比例對於其碎形維度與幾何外觀並無顯著的影響。三維擴散決定聚集也將一併討論。最後，我們採用中心移動的方法來模擬，我們發現 Eden 模型與 Witten-Sander 模型間的關聯可由此法建立起來。中心移動模擬結果的分析可以提供我們瞭解擴散決定聚集的線索。